Polarization-Dependent Loss (PDL) Measurement Theory

This document provides a detailed summary of the theoretical basis of the 4-polarization state method for PDL measurement (also referred to as the Mueller-Stokes method). It relates specifically to the swept-wavelength system (SWS) which employs this method for calculating PDL values from insertion-loss measurements, and proves the validity of the 4-state method for PDL calculations. It is a record of the fundamental equations that are incorporated into the SWS-PDL software suite. This method is described and approved as IEC61300–3–12.

PDL 4-State Method Derivation

Mueller calculus

Mueller calculus is a mathematical tool to deal with polarization that is similar to the Jones method in that the effect of an optical element on the polarization of light is treated as a matrix operation. More specifically, the transmission of light through an arbitrary device can be expressed as the product of the Mueller matrix of the device under test (DUT) (a 4x4 matrix) and the input Stokes vector (1x4 matrix). The result is another Stokes vector. The details of this mathematical operation are reviewed in detail in (ref. 1, pg. 83).

The transmission equations are as follows:

$$\begin{bmatrix} T_0 \\ T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} m_{00} & m_{01} & m_{02} & m_{03} \\ - & - & - & - \\ - & - & - & - \\ - & - & - & - \end{bmatrix} \begin{bmatrix} S_0 \\ S_0 \cos\omega\cos\alpha \\ S_0 \cos\omega\sin\alpha \\ S_0 \sin\omega \end{bmatrix} \qquad [1]$$

T_0 is the intensity of the transmitted or output power. T^1, T^2, and T^3 relate to the polarization state of the output power. Note that these values are not measured by a standard detector. The first row of the Mueller matrix contains all of the terms that relate to the power transmission. The remaining terms are not involved in the determination of PDL and thus have been omitted. The input Stokes vector is in a general form for an elliptically polarized light. S_0 is the intensity of the input power, α is the angle of the azimuth of the input polarization, and ω is the ellipticity of the input polarization.

Determining PDL

The PDL of the DUT is determined from the difference between the minimum and maximum values for the transmitted intensity of the variable T_0. To calculate this, we must solve equation 1 and take the partial transmitted derivatives with respect to the polarization variables α and ω, and set the resulting equations to 0, as follows:

$$T_0 = m_{00}S_0 + m_{01}S_0\cos\omega\cos\alpha + m_{02}S_0\cos\omega\sin\alpha + m_{03}S_0\sin\omega \quad [1.1]$$

$$\frac{\partial T_0}{\partial\alpha} = -m_{01}S_0\cos\omega\sin\alpha + m_{02}S_0\cos\omega\cos\alpha \quad [1.2a]$$

$$\frac{\partial T_0}{\partial\omega} = -m_{01}S_0\sin\omega\cos\alpha - m_{02}S_0\sin\omega\sin\alpha + m_{03}S_0\cos\omega \quad [1.3a]$$

then set 1.2a and 1.2b to 0:

$$m_{01}\sin\alpha = m_{02}\cos\alpha \quad \text{or} \quad \alpha = \tan^{-1}\left(\frac{m_{02}}{m_{01}}\right) + n\pi \quad [1.2b]$$

$$\omega = \tan^{-1}\left(\frac{m_{03}}{m_{01}\cos\alpha + m_{02}\sin\alpha}\right) \quad [1.3b]$$

where n=0,1 in order to account for the fact that the azimuth variable has a range from 0 to 2π. Equation 1.2b expresses the α variable in terms of the Mueller elements only. It can then be used to eliminate α from equation 1.3b. To do so, the following manipulation of the first form of 1.2b is required.

$$\frac{\sin\alpha}{\cos\alpha} = \frac{m_{02}}{m_{01}}$$

$$\sin^2\alpha = \left(\frac{m_{02}}{m_{01}}\right)^2\cos^2\alpha \qquad \cos^2\alpha = \left(\frac{m_{01}}{m_{02}}\right)^2\sin^2\alpha$$

then using $\sin^2\alpha + \cos^2\alpha = 1$, we get:

$$\sin^2\alpha = \left(\frac{m_{02}}{m_{01}}\right)^2\left(1-\sin^2\alpha\right) \qquad \cos^2\alpha = \left(\frac{m_{01}}{m_{02}}\right)^2\left(1-\cos^2\alpha\right)$$

$$\sin\alpha = \frac{m_{02}}{\sqrt{m_{01}^2 + m_{02}^2}} \qquad \cos\alpha = \frac{m_{01}}{\sqrt{m_{01}^2 + m_{02}^2}} \quad [2]$$

Equations 2 are then substituted into the denominator of 1.3b to yield ω in terms of only the Mueller matrix elements as follows.

$$\omega = \tan^{-1}\left[\frac{m_{03}}{\frac{m_{01}{}^2}{\sqrt{m_{01}{}^2 + m_{02}{}^2}} + \frac{m_{02}{}^2}{\sqrt{m_{01}{}^2 + m_{02}{}^2}}}\right] \quad [3a]$$

which simplifies to the following:

$$\omega = \tan^{-1}\left(\frac{m_{03}}{\sqrt{m_{01}{}^2 + m_{02}{}^2}}\right) \quad [3b]$$

Equations 3b and 1.2b give us the variables ω in terms of only the Mueller elements. They can be manipulated to form the trigonometric expressions in equation 1 for the case of the maximum and minimum transmission. The maximum value is the result for the case where n=0 from equation 1.2b. Similarly n=1 relates to the minimum value. Therefore, we require expressions for cosωcosα, cosωsinα and sinω from equations 1.2b and 3n. Sinα and cosα are already available from equation 2.

#1 From equation 3b:

$$\sin\omega = \cos\omega\left(\frac{m_{03}}{\sqrt{m_{01}{}^2 + m_{02}{}^2}}\right)$$

$$\sin^2\omega = \left(1 - \sin^2\omega\right)\left(\frac{m_{03}{}^2}{m_{01}{}^2 + m_{02}{}^2}\right)$$

$$\therefore \quad \sin\omega = \frac{m_{03}}{\sqrt{m_{01}{}^2 + m_{02}{}^2 + m_{03}{}^2}} \quad [4]$$

#2 Also from equation 3b:

$$\cos\omega = \sin\omega\left(\frac{\sqrt{m_{01}{}^2 + m_{02}{}^2}}{m_{03}}\right)$$

$$\downarrow$$

$$\cos^2\omega = \left(1 - \cos^2\omega\right)\left(\frac{m_{01}{}^2 + m_{02}{}^2}{m_{03}{}^2}\right)$$

$$\downarrow$$

$$\therefore \quad \cos\omega = \frac{\sqrt{m_{01}{}^2 + m_{02}{}^2}}{\sqrt{m_{01}{}^2 + m_{02}{}^2 + m_{03}{}^2}} \quad [5]$$

We can simplify the above expressions by using the substitutions: $a = \sqrt{{m_{01}}^2 + {m_{02}}^2 + {m_{03}}^2}$

Thus, the expression for the maximum transmission for arbitrary elliptically polarized light becomes:

$$\begin{bmatrix} T_0 \\ T_1 \\ T_2 \\ T_3 \end{bmatrix}_{MAX} = \begin{bmatrix} m_{00} & m_{01} & m_{02} & m_{03} \\ - & - & - & - \\ - & - & - & - \\ - & - & - & - \end{bmatrix} \begin{bmatrix} S_0 \\ S_0 m_{01} / a \\ S_0 m_{02} / a \\ S_0 m_{03} / a \end{bmatrix} \qquad [6]$$

where the resulting solution is:

$$T_{0MAX} = S_0 m_{00} + \frac{S_0 {m_{01}}^2}{a} + \frac{S_0 {m_{02}}^2}{a} + \frac{S_0 {m_{03}}^2}{a}$$

that can be simplified to the following: $T_{o\max} = S_0 [m_{00} + a]$ [7]

To find the minimum transmission, we follow the same path leading to equation 7, but with n=1.

This results in equation 1.2b having the functional form of $\alpha = \alpha_0 + \pi$. For this expression, we can make use of the following trigonometric expressions:

$$\cos(\alpha_0 + \pi) = -\cos(\alpha_0)$$
$$\sin(\alpha_0 + \pi) = \sin(\alpha_0)$$

these relationships can be used to solve for equation 1.3b with n=1 resulting in:

$$\omega = \tan^{-1}\left(\frac{m_{03}}{-(m_{01}\cos\alpha + m_{02}\sin\alpha)} \right) \qquad [8]$$

when equation 8 is used instead of equation 1.3b. Repeating all of the steps leading to the maximum transmission (equations 2 to 6), we arrive at the condition for minimum transmission:

$$\begin{bmatrix} T_0 \\ T_1 \\ T_2 \\ T_3 \end{bmatrix}_{MIN} = \begin{bmatrix} m_{00} & m_{01} & m_{02} & m_{03} \\ - & - & - & - \\ - & - & - & - \\ - & - & - & - \end{bmatrix} \begin{bmatrix} S_0 \\ -S_0 m_{01} / a \\ -S_0 m_{02} / a \\ -S_0 m_{03} / a \end{bmatrix} \qquad [9]$$

which, like 7a, simplifies to : $T_{0\min} = S_0 [m_{00} - a]$ [7b]

At this point, we can determine the PDL from equations 7a and 7b and using the PDL definition:

$$PDL = -10 \times Log\left(\frac{T_{0\min}}{T_{0\max}}\right) = -10 \times Log\left(\frac{m_{00} - \sqrt{m_{01}^{2} + m_{02}^{2} + m_{03}^{2}}}{m_{00} + \sqrt{m_{01}^{2} + m_{02}^{2} + m_{03}^{2}}}\right) \quad [10]$$

Equation 10 illustrates that in order to determine the PDL of a device, we must determine the first row of its Mueller matrix. The four matrix elements can be determined by applying four different polarization states to the device. The resulting set of four equations can be solved to find the four unknown Mueller elements, provided that the 4 input polarization states are linearly independent. The four independent vectors that we employ are as follows, with their respective Stokes vectors:

1) horizontal linear polarized light $S_{(0°)} = [S_0 \quad S_0 \quad 0 \quad 0]$
2) vertical linear polarized light $S_{(90°)} = [S_0 \quad -S_0 \quad 0 \quad 0]$
3) 45° linear polarized light $S_{(45°)} = [S_0 \quad 0 \quad S_0 \quad 0]$
4) left-hand circularly polarized light $S_{(LHC)} = [S_0 \quad 0 \quad 0 \quad -S_0]$ [11]

If we use these four polarization states with equation 1, we get the transmission for each state as follows:

$$T_{0(0°)} = S_0[m_{00} + m_{01}] \qquad T_{0(90°)} = S_0[m_{00} - m_{01}]$$
$$T_{0(45°)} = S_0[m_{00} + m_{02}] \qquad T_{0(LHC)} = S_0[m_{00} - m_{03}] \quad [12]$$

When we algebraically manipulate the above equations and normalize the intensities, we will get the Mueller elements as a function of the four measurable intensities, which is what we need to finish solving equation 10 and determine PDL.

$$m_{00} = \frac{T_{0(0°)} + T_{0(90°)}}{2} \qquad m_{01} = \frac{T_{0(0°)} - T_{0(90°)}}{2}$$

$$m_{02} = T_{0(45°)} - \left(\frac{T_{0(0°)} + T_{0(90°)}}{2}\right) \qquad m_{03} = \frac{T_{0(0°)} + T_{0(90°)}}{2} - T_{0(LHC)} \quad [13]$$

Wavelength dependencies

Equation 13 gives the correct expressions for the Mueller elements needed to solve for the PDL by equation 10 when the four input polarization states are exactly as stated in equations 11. In practice, however, the circularly polarized light is usually elliptically polarized. This is a result of the fact that circularly polarized light is usually made by an arrangement of a linear polarizer and a quarter waveplate with its required axis arranged at 45° to the polarizer's axis (ref. 4 pg. 294). The quarter waveplate only has its required π/2 radians of retardation at a particular wavelength. At other wavelengths, the circular polarizer will produce elliptically polarized light. Thus, we will need to apply a correction to equations 11, 12, and 13 to account for the wavelength dependencies since the major applications (PS3 meter, SWS) work at multiple wavelengths.

To begin, we require a Stokes vector for the output of a circular polarizer at a wavelength that is different form its center value. The retardation of a quarter waveplate varies with wavelength, to a first-order approximation, as follows:

$$\delta = \frac{\pi CW}{2\lambda} \text{ for QWP and } \delta = \frac{\pi CW}{\lambda} \text{ for HWP} \qquad [14]$$

where CW is the center wavelength of the waveplate and λ is the variable wavelength. Then, the Mueller matrix for a linear retarder at an arbitrary angle θ with retardation δis as follows:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C_2{}^2 + S_2{}^2\beta & C_2S_2(1-\beta) & -S_2\mu \\ 0 & C_2S_2(1-\beta) & S_2{}^2 + C_2{}^2\beta & C_2\mu \\ 0 & S_2\mu & -C_2\mu & \beta \end{bmatrix} \qquad [15]$$

where the following abbreviations have been used: C2=cos2θ, S2=sin2θ, μ=sinδ, β=cosδ. For the case of a circular polarizer, the angle θ is +45°. With this, we can simplify 15 to:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \beta & 0 & -\mu \\ 0 & 0 & 1 & 0 \\ 0 & \mu & 0 & \beta \end{bmatrix} \qquad [15b]$$

From 15b we can now calculate the Stokes vector of the wavelength-dependent elliptically polarized light by inputting the linear polarized light from the polarizer. The matrix calculation is as follows:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \beta & 0 & -\mu \\ 0 & 0 & 1 & 0 \\ 0 & \mu & 0 & \beta \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -\beta \\ 0 \\ \mu \end{bmatrix} = \begin{bmatrix} 1 \\ -\cos\delta \\ 0 \\ -\sin\delta \end{bmatrix} \qquad [16]$$

These results can now be used to replace the $S_{(LHC)}$ from equation 11. This leads to the change in the expression for $_{T0(LHC)}$ in equation 12 to become:

$$T_{0(LHE)} = S_0\left(m_{00} - m_{01}\cos\delta - m_{03}\sin\delta\right) \qquad [17]$$

where $T_{0(LHE)}$ is the transmitted intensity for the elliptically polarized light. When we algebraically manipulate equation 17 along with the remaining three unchanged equations from 12, we once again generate the expressions for the Mueller elements but now as a function of wavelength.

$$m_{00} = \frac{T_{0(0°)} + T_{0(90°)}}{2} \qquad m_{01} = \frac{T_{0(0°)} - T_{0(90°)}}{2}$$

$$m_{02} = T_{0(45°)} - \left(\frac{T_{0(0°)} + T_{0(90°)}}{2}\right) \qquad m_{03} = \frac{-T_{0(LHE)} + m_{00} - m_{01}\cos(\delta)}{\sin(\delta)} \qquad [18]$$

References

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